Estimating Parameters

and

Mean Confidence Interval

What is a **Parameter**?

Parameter is any numerical measurement related to a population.

What are some common **Parameters**?

Here are some common parameters:

- Population **Proportion** p
- Population **Mean** μ
- Population **Standard Deviation** σ

What do we need to start the **Estimation** process?

We must have a randomly selected sample from the population that has the correct point-estimate.

What is a **Point-Estimate**?

In statistic, the **Point-Estimate** is an **Estimator** of some **Parameter** of the population.

Point-Estimate is calculated from the sample data and it is served as a the **Best-Guess** for our estimation of the parameter.

What is a **Confidence Interval**?

In statistics, a **Confidence Interval** is a range of values computed from the statistics of the observed data, that might contain the true value of a population parameter.

Every **Confidence Interval** comes with a **Confidence Level**.

What is a **Confidence Level**?

Confidence Level represents the probability that the true parameter lies within the confidence interval.

Confidence Level is usually expressed as a percentage.

What are some common **Confidence Levels**?

Here are some common confidence levels:

- 90%
- 95%
- 99%

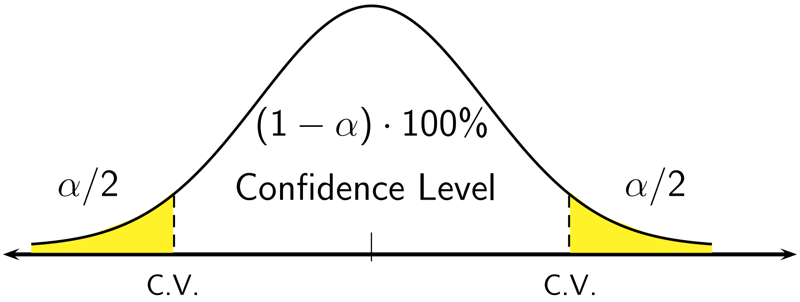
Important information about **Confidence Levels**:

- When confidence level is not given, use 95%.
- For significance level α, where $0 < \alpha < 1$ the confidence level is $(1 - \alpha) \cdot 100\%$.

Confidence Level vs. Significance Levels Chart:

Confidence Level	Significance Level
90%	$\alpha = 0.1$
95%	$\alpha = 0.5$
99%	$\alpha = 0.01$
$(1-\alpha)\cdot 100\%$	$\alpha,\ 0 < \alpha < 1$

Confidence Level vs. **Significance Level** Display:



Confidence Interval for Population Mean:

- Final Answer: $\cdots < \mu < \cdots$
- General Format: $\bar{x} - E < \mu < \bar{x} + E$
- Sample Mean: $\bar{x} = \dfrac{\sum x}{n}$ with sample size n
- Critical Value & Margin of Error:

Case I: σ known	Case II: σ unknown
CV: $Z_{\alpha/2}$	CV: $t_{\alpha/2}$ with $df = n - 1$
Error: $E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	Error: $E = t_{\alpha/2} \cdot \dfrac{s}{\sqrt{n}}$

Example:

In a survey of 54 students in college, their mean age was 32.3 years. It is known that standard deviation of ages of all college students is 8.9 years.

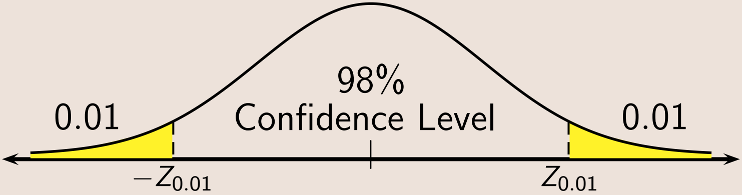
- Identify all information provided, using mathematical symbols.
- Find the critical value for constructing the 98% confidence interval for the mean age of all college students.
- Find the margin of error when constructing a 98% confidence interval for the mean age of all college students.
- Find the 98% confidence interval for the mean age of all college students.

Solution:

- Identify all information provided, using mathematical symbols.

  $n = 54$, $\bar{x} = 32.3$, and $\sigma = 8.9$.

- Find the critical value for constructing the 98% confidence interval for the mean age of all college students.



Since we know σ, we are in case I.

$Z_{0.05} = \textbf{invNorm}(0.99, 0, 1) = 2.326$

Solution Continued:

- Find the margin of error when constructing a 98% confidence interval for the mean age of all college students.

  Using case I with $n = 54$, $\sigma = 8.9$, and $Z_{\alpha/2} = 2.326$, we get

  $$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2.326 \cdot \frac{8.9}{\sqrt{54}} \approx 2.8$$

- Find the 98% confidence interval for the mean age of all college students.

  With $\bar{x} = 32.3$,

  $$\begin{aligned} \bar{x} - E &< \mu < & \bar{x} + E \\ 32.3 - 2.8 &< \mu < & 32.3 + 2.8 \\ 29.5 &< \mu < & 35.1 \end{aligned}$$

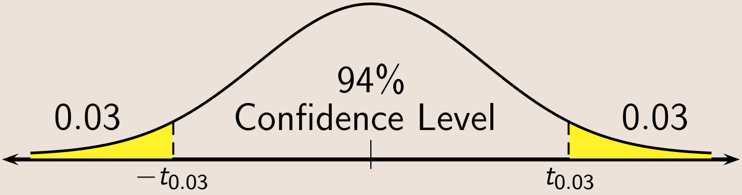
Example:

In a survey of 12 students in college, they had a mean monthly income of \$1650 with standard deviation of \$250.

- Identify all information provided, using mathematical symbols.
- Find the critical value for constructing the 94% confidence interval for the mean monthly income of all college students.
- Find the margin of error when constructing a 94% confidence interval for the mean monthly of all college students.
- Find the 94% confidence interval for the mean monthly of all college students.

Solution:

- Identify all information provided, using mathematical symbols.
  $n = 12$, $\bar{x} = 1650$, and $s = 250$. it is worth nothing that population standard deviation σ is unknown.
- Find the critical value for constructing the 94% confidence interval for the mean monthly income of all college students.



Since we do not know σ, we are in case II with
$df = n - 1 = 12 - 1 = 11$

$t_{0.03} = \textbf{invT}(0.97, 11) = 2.096$

Solution Continued:

- Find the margin of error when constructing a 94% confidence interval for the mean monthly of all college students.

  Using case II with $n = 12$, $s = 250$, and $t_{\alpha/2} = 2.098$, we get

  $$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.098 \cdot \frac{250}{\sqrt{12}} \approx 151$$

- Find the 98% confidence interval for the mean monthly income of all college students.

  With $\bar{x} = 32.3$,

  $$\begin{aligned} \bar{x} - E &< \mu < && \bar{x} + E \\ 1650 - 151 &< \mu < && 1650 + 151 \\ 1499 &< \mu < && 1801 \end{aligned}$$

Finding $\bar{x}$ & E from Confidence Interval:

Given the confidence interval **Lower** $<\mu<$ **Upper**, then

- $\bar{x} = \dfrac{\textbf{Upper Value} + \textbf{Lower Value}}{2}$
- $E = \dfrac{\textbf{Upper Value} - \textbf{Lower Value}}{2}$

Mean Confidence Interval & TI:

Here are the steps on TI when constructing confidence interval for population proportion:

- STAT
- TESTS
- ZInterval when σ is known
- TInterval when σ is unknown

Pay close attention to the following:

- When confidence level is not given, use 95%.
- Always round your final answer to consistent with $\bar{x}$, and use mathematical notation to display your final answer.

Example:

In a sample of 40 textbooks at the college bookstore, the mean price was \$89.75. It is also reported that the standard deviation of prices of all textbooks is \$22.65.

- Identify all information provided, using mathematical symbols.
- Find the confidence interval for the mean price of all textbooks.
- Find the margin of error.

Solution:

- Identify all information provided, using mathematical symbols.

  $n = 40$, $\bar{x} = 89.75$, $\sigma = 22.65$

Solution Continued:

- Find the confidence interval for the mean price of all textbooks.

  Since we have σ, we would use **ZInterval** for constructing confidence interval, and since the confidence level is not given, we use 95% confidence level.
  Following the TI commands **STAT** $>$**TESTS** $>$ **ZInterval** with $\bar{x} = 89.75$, $n = 40$, $\sigma = 22.65$, and **C-Level:** 0.95, we get

$$82.73 < \mu < 96.77$$

- Find the margin of error.

$$E = \frac{\textbf{Upper Value} - \textbf{Lower Value}}{2} = \frac{96.77 - 82.73}{2} = 7.02$$

Example:

Scores of 10 randomly selected exams had the following scores.

78	83	90	65	100
95	58	80	72	70

- Find the mean and standard deviation of these randomly selected exams. Round your final answer to a whole number.
- Find the 99% confidence interval for the mean of all such exams.
- Find the margin of error.

Solution:

- Find the mean and standard deviation of these randomly selected exams. Round your final answer to a whole number.

  Using TI calculator, we get $\bar{x} = 79$, $s = 13$.

Solution Continued:

- Find the 99% confidence interval for the mean of all such exams.

  Since we do not know σ, we would use **TInterval** for constructing confidence interval.
  Following the TI commands **STAT** $>$**TESTS** $>$ **TInterval** with $\bar{x} = 79$, $n = 10$, $s = 13$, and **C-Level:** 0.99, we get

$$66 < \mu < 92$$

- Find the margin of error.

$$E = \frac{\textbf{Upper Value} - \textbf{Lower Value}}{2} = \frac{92 - 66}{2} = 13$$